is consistent with the physics of the magnetization in a ferrosuspension. For an applied field the magnetization of the ferrosuspension in the direction of the field instantly (for $\tau_1 = 0$) takes the value m_{10} because of Néel relaxation. But over a time of order τ_2 the magnetization of the ferrosuspension increases to the value m_{20} because of Brownian relaxation. In the equations given here, terms involving the coefficient K_3 in the equation of state (3.3) correspond to the effect of internal fields on the magnetization m_1 and m_2 due to the existence of magnetizations $\mathbf{m}_1$ and $\mathbf{m}_2$, respectively. This recalls the situation in antiferromagnets where there are also two effective fields resulting from the magnetizations $\mathbf{m}_1$ and $\mathbf{m}_2$ of the sublattices.

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EVOLUTION EQUATION FOR THE VORTEX DISTRIBUTION FUNCTION IN THE PLANAR CASE

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In [1-5] a system of linear vortex lines in an ideal fluid is used as a model of two-dimensional turbulence. There are examples which support the plausibility of this model; in particular, results have been obtained with the model which are interesting from the point of view of the statistical theory of turbulence [1], dynamical meteorology [4], and numerical modeling of the streamlining of bodies at large Reynolds numbers [5].

In the above papers the system of vortices was studied in a state of statistical equilibrium. But in real hydrodynamic turbulence, nonequilibrium states of the fluid are important as well, where the evolution is characterized by statistical irreversibility. It is therefore of interest to consider nonequilibrium evolution in model systems by the methods of kinetic theory [7, 8]. Some asymptotic solutions of the BBGKY hierarchy for a system of linear vortices have been considered in [1].

In the present paper, the nonequilibrium statistical properties of this model are studied using the Liouville equation for an ensemble of vortex lines. Analysis and summation of the formal time-dependent perturbation series are carried out with the help of the dia-

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grammatic techniques of Prigogine and Balescu. In order to suppress the divergence at large values of the time, the renormalization procedure is used. We obtain a closed evolution equation for the vortex distribution function. The equation contains both the convective Helmholtz operator and a quasilinear elliptic operator of second order with nonlocal coefficients. The equation takes into account explicitly the physical effect of variable sign in the vortex viscosity, because the local sign of the dissipative matrix is determined by the instantaneous vortex distribution. It is shown that in the absence of internal boundaries, the evolution everywhere goes in the direction of increasing informational entropy and the vortex distribution approaches a stationary distribution.

1. In the infinite x, y plane, we consider a system of N point vortices with identical circulation strengths $\varkappa$. The Cartesian coordinates of the vortices (x_i, y_i), $i = 1, 2, \ldots, N$ are canonically conjugate with respect to the Hamiltonian [6]

$$H_N = -\sum_{i<j}^{N} V_{ij}(|\mathbf{r}_i - \mathbf{r}_j|),\ V_{ij} = \ln|\mathbf{r}_i - \mathbf{r}_j|.$$

The dynamical equations for the system of vortices are

$$d\mathbf{r}_i/dt = -\varkappa \mathbf{e} \times \nabla_i H_N, \quad i = 1, \ldots, N, \tag{1.1}$$

where $\mathbf{e}$ is a unit vector normal to the plane of motion of the vortices.

Let $f_N(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N, t)$ be the joint probability density distribution for the system of N vortices. We have the normalization condition

$$\int d\mathbf{r}_1 \ldots d\mathbf{r}_N f_N = 1. \tag{1.2}$$

A statistical description of the motion of the system equivalent to (1.1) is given by the Liouville equation [7] which we will use in one of two forms:

$$\frac{\partial f_N}{\partial t} = -\varkappa \mathbf{e} \times \sum_{i<j}^{N} \nabla_i V_{ij} \cdot (\nabla_i - \nabla_j) f_N = -\varkappa \sum_{i<j}^{N} L_{ij} f_N \equiv -\varkappa L f_N; \tag{1.3}$$

$$\frac{\partial f_N}{\partial t} = \varkappa \sum_{i<j}^{N} [\{V_{ij}, f_N\}_i + \{V_{ij}, f_N\}_j] \equiv \varkappa \{H_N, f_N\}. \tag{1.4}$$

Here $\{V_{ij}, f_N\}_i$ are the Poisson brackets calculated with respect to (x_i, y_i).

We will assume that the total vorticity in the fluid is equal to zero; this is analogous to the quasineutrality condition in a plasma [7]. Then the first two BBGKY equations obtained from (1.4) have the form

$$\frac{\partial F_1(\mathbf{r}_1, t)}{\partial t} = \varkappa c\Big[\{\psi(\mathbf{r}_1, t), F_1(\mathbf{r}_1, t)\} + \int d\mathbf{r}_2 \{V_{12}, F_2(\mathbf{r}_1, \mathbf{r}_2, t) - F_1(\mathbf{r}_1, t) F_1(\mathbf{r}_2, t)\}\Big]; \tag{1.5}$$

$$\begin{aligned}\frac{\partial F_2(\mathbf{r}_1, \mathbf{r}_2 t)}{\partial t} &= \varkappa\{H_2, F_2\} + \varkappa c [\{\psi(\mathbf{r}_1, t) + \psi(\mathbf{r}_2, t), F_2(\mathbf{r}_1, \mathbf{r}_2, t)\} \\ &+ \int d\mathbf{r}_3 \{V_{13} + V_{23}, F_3(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, t) - F_1(\mathbf{r}_3, t) F_2(\mathbf{r}_1, \mathbf{r}_2, t)\}\Big].\end{aligned} \tag{1.6}$$

Here c is the average vortex density, $\varkappa c F_1(\mathbf{r}, t)$ physically is a local vorticity, defined in terms of the statistical ensemble $f_N(t)$ [1, 4]. In our case the vorticity for a system of point vortices has the form

$$\omega(\mathbf{r}, t) = \varkappa \sum_{i=1}^{N} \delta(\mathbf{r} - \mathbf{r}_i),\ F_1(\mathbf{r}, t) = \Omega^{-1} \int d\mathbf{r}_2 \ldots d\mathbf{r}_N f_N(\mathbf{r}, \mathbf{r}_2, \ldots, \mathbf{r}_N, t).$$

Therefore, in the limit $N, \Omega \to \infty$, $N\Omega^{-1} \to c$

$$\langle \omega(\mathbf{r}, t) \rangle = \lim_{N,\Omega\to\infty} \varkappa \int d\mathbf{r}_1 \ldots d\mathbf{r}_N \sum_{i=1}^{N} \delta(\mathbf{r} - \mathbf{r}_i) f_N(\mathbf{r}_1, \ldots, \mathbf{r}_N, t) = \lim_{N,\Omega\to\infty} \varkappa N\Omega^{-1} F_1(\mathbf{r}, t) = \varkappa c F_1(\mathbf{r}, t).$$

The function

$$\psi(\mathbf{r},t)=\int d\mathbf{r}_2V_{12}n(\mathbf{r}_2,t)$$

can be determined in terms of the factor $n(\mathbf{r}, t) = F_1(\mathbf{r}, t)-1$.

The Hamiltonian H_N contains only terms of the form V_{ij} so that the system of vortices is strongly interacting. Thus, in (1.5) we have $\{H_1, F_1\} \equiv 0$ and this complicates the construction of a closed evolution equation for F_1 by the usual methods [7, 8]. In [1] the analysis of the BBGKY equations for point vortices was based on a formal asymptotic expansion in a small parameter, which was introduced as a singularity in the first term of the equation for F_s with $s \geqslant 2$ (cf. (1.6)). The derivation of a closed equation for $F_1(\mathbf{r}, t)$ was not discussed.

2. The derivation of closed kinetic equations in strongly interacting systems is usually based on various assumptions [9, 10], which allow one to isolate the essential features of the required equations and serve as a basis for doing a formal asymptotic analysis. The results are then tested on a real system.

In our case, because $\varkappa cF_1(\mathbf{r}, t)$ is the average local vortex density in turbulent flow, physically the equation for F_1 should contain the convective Helmholtz operator [11].

It is easily shown (cf. [12]) that in the limit $\varkappa \to 0$, $c \to \infty$ when the average vorticity is bounded ($\varkappa c \to O(1)$) Eq. (1.6) and the succeeding equations of the chain are satisfied identically by distributions of the form

$$F_s=\prod_{i=1}^{s}F_1(\mathbf{r}_i,t),$$

if F_1 satisfies the equation

$$dF_1(\mathbf{r},t)/dt=\varkappa c\{\psi(\mathbf{r},t),F_1(\mathbf{r},t)\}\equiv-\mathbf{U}(\mathbf{r},t)\cdot\nabla F_1(\mathbf{r},t), \tag{2.1}$$

where

$$\mathbf{U}(\mathbf{r},t)=\varkappa c\mathbf{e}\times\nabla\cdot\int d\mathbf{r}_1V(|\mathbf{r}-\mathbf{r}_1|)\,n(\mathbf{r}_1,t)$$

is the average hydrodynamic velocity induced by the locally noncanceling vorticity. Then (2.1) has the form of a self-consistent field equation [7] and (within the uniform factor $\varkappa c$) reduces to the Helmholtz equation for the vortex field in the planar case.

Using the above remarks we assume the following asymptotic relations between the parameters of the system:

$$\varkappa c\sim O(1),\quad \varkappa\sim O(\varepsilon),\quad \varepsilon\ll 1. \tag{2.2}$$

Using the Bogolyubov method with (2.2) one can obtain a sequence of closed equations for F_1 in which (2.1) will be the zeroth-order equation.

Indeed, following [7], we introduce the expansion

$$F_s(\mathbf{r}_1,\ldots,\mathbf{r}_s,t)=F_s^{(0)}(\mathbf{r}_1,\ldots,\mathbf{r}_s,F_1)+\varkappa F_s^{(1)}(\mathbf{r}_1,\ldots,\mathbf{r}_s,F_1)+\ldots,s\geqslant 2, \tag{2.3}$$

where it is assumed that $F_s^{(i)}$ only depends on time implicity through the function $F_1(\mathbf{r}, t)$.

We look for an evolution equation for F_1 in the form

$$\partial F_1(\mathbf{r},t)/\partial t=L_0(F_1)+\varkappa L_1(F_1)+\ldots. \tag{2.4}$$

Then from (1.5) and (2.3) it follows that

$$L_0(F_1)=\varkappa c\Big[\{\psi(\mathbf{r}_1,t),F_1(\mathbf{r}_1,t)\}+\int d\mathbf{r}_2\{V_{12},F_2^{(0)}(\mathbf{r}_1,\mathbf{r}_2,F_1)-F_1(\mathbf{r}_1,t)F_1(\mathbf{r}_2,t)\}\Big], \tag{2.5}$$

$$L_1(F_1)=\varkappa c\int d\mathbf{r}_2\{V_{12},F_2^{(1)}(\mathbf{r}_1,\mathbf{r}_2,F_1)\}\ldots.$$

The operation of differentiation of F_s with respect to time can be written with the help of (2.3) and (2.4) in operator series form

$$\frac{\partial F_s}{\partial t} = \frac{\delta F_s}{\delta F_1}\frac{\partial F_1}{\partial t} = D_0 F_s^{(0)} + \varkappa\left(D_1 F_s^{(0)} + D_0 F_s^{(1)}\right) + \ldots, s \geqslant 2, \tag{2.6}$$

where $L_0(F_1)$ appears in operator D_0, $L_1(F_1)$ in operator D_1, etc.

Substitution of (2.3) and (2.6) in (1.6) gives to zero order in $\varkappa$:

$$D_0 F_2^{(0)}(\mathbf{r}_1, \mathbf{r}_2, F_1) = \varkappa c\Big[\{\psi(\mathbf{r}_1, t) + \psi(\mathbf{r}_2, t), F_2^{(0)}(\mathbf{r}_1, \mathbf{r}_2, F_1)\} + \int d\mathbf{r}_3\,\{V_{13} + V_{23}, F_3^{(0)}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, F_1) - F_1(\mathbf{r}_3, t)\; F_2^{(0)}(\mathbf{r}_1, \mathbf{r}_2, F_1)\}\Big].$$

It can be verified directly that for

$$F_3^{(0)}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, F_1) = \prod_{i=1}^{3} F_1(\mathbf{r}_i, t)$$

the solution of (2.7) will be

$$F_2^{(0)}(\mathbf{r}_1, \mathbf{r}_2, F_1) = \prod_{i=1}^{2} F_1(\mathbf{r}_i, t).$$

And in view of the definition of D_0 and (2.5) we have

$$D_0 F_2^{(0)}(\mathbf{r}_1, \mathbf{r}_2, F_1) = F_1(\mathbf{r}_1, t)\, L_0(\mathbf{r}_2, t) + F_1(\mathbf{r}_2, t)\, L_0(\mathbf{r}_1, t) = F_1(\mathbf{r}_1, t)\,\varkappa c \times$$
$$\times\{\psi(\mathbf{r}_2, t), F_1(\mathbf{r}_2, t)\} + F_1(\mathbf{r}_2, t)\,\varkappa c\,\{\psi(\mathbf{r}_1, t), F_1(\mathbf{r}_1, t)\},$$

from which it follows that this form for $F_2^{(0)}$ reduces (2.7) to an identity. At the same time, substitution of the solution of (2.7) in the zeroth-order equation (2.4) gives (2.1).

Thus, relations (2.2) between the parameters of the system lead, at least in the zeroth-order approximation, to a reasonable physical result, and this provides some justification for their use in higher order approximations.

The procedure used here is formally identical to the method of Bogolyubov in a plasma. It is well known that for a spatially nonuniform system of this kind, the Bogolyubov method leads to extremely unwieldy calculations in the higher orders, and it is necessary to introduce additional assumptions. Therefore, we use below the equivalent method of Prigogine and Balescu which has the advantage that it allows one to specify the orders of the characteristic times over which the different terms of the required equations are important. In [9, 10] this method was applied to a strongly interacting system.

3. We use the Prigogine and Balescu method in the resolvent formalism. The notation used is the same as in [8]. The starting point is a Fourier analysis of the class of functions periodic inside a square of area Ω. The N-vortex distribution function f_N and the vortex interaction potential V_{ij} are expanded in Fourier series

$$f_N(\mathbf{r}_1, \ldots, \mathbf{r}_N, t) = \Omega^{-N}\left[\rho_0 + \frac{4\pi^2}{\Omega}\sum_{j=1}^{N}\sum_{\mathbf{k}}{}' \rho(\mathbf{k}, t)\, e^{i\mathbf{k}\mathbf{r}_j} + \ldots\right]; \tag{3.1}$$

$$V(|\mathbf{r}_m - \mathbf{r}_j|) = 4\pi^2\Omega^{-1}\sum_{\mathbf{l}} V(l)\, e^{i\mathbf{l}(\mathbf{r}_m - \mathbf{r}_j)}, \tag{3.2}$$

where $k_j = 2\pi\Omega^{-1/2}\mathbf{n}_j$, and $\mathbf{n}_j$ is a vector whose magnitude is an integer.

The normalization condition (1.2) gives $\rho_0 = 1$. The Fourier coefficients in expansion (3.1) are related to the reduced distribution functions. In particular

$$f(\mathbf{r}, t) = cF_1(\mathbf{r}, t) = c\left(1 + \int d\mathbf{k}\, e^{i\mathbf{k}\mathbf{r}}\rho(\mathbf{k}, t)\right).$$

With the help of (3.1) and (3.2), one can obtain a Fourier representation of the formal solution of the Liouville equation (1.3) in the form of a perturbation series:

$$\rho(\{\mathbf{k}\}, t) = (2\pi)^{-1}\oint dz\, e^{-izt}\sum_{n=0}^{\infty}(-\varkappa)^n\sum_{\{\mathbf{k}'\}}\left(\frac{4\pi^2}{\Omega}\right)^{\nu'-\nu}\langle\{\mathbf{k}\}|\, R_0(z)\,[LR_0(z)]^n\,|\{\mathbf{k}'\}\rangle\, \rho(\{\mathbf{k}'\}, 0). \tag{3.3}$$

The matrix elements in (3.3) are defined using plane-wave basis states. In particular

$$\langle\{\mathbf{k}\}|L|\{\mathbf{k}'\}\rangle = \sum_{n<j}\langle\{\mathbf{k}\}|L_{nj}|\{\mathbf{k}'\}\rangle, \tag{3.4}$$

$$\langle\{\mathbf{k}\}|L_{nj}|\{\mathbf{k}'\}\rangle = 4\pi^2\Omega^{-1}\mathbf{e}\times i(\mathbf{k}_n'-\mathbf{k}_n)V(|\mathbf{k}_n'-\mathbf{k}_n|)\,i(\mathbf{k}_n'-\mathbf{k}_j')\,\delta_{\mathbf{k}_n'+\mathbf{k}_j'-\mathbf{k}_n-\mathbf{k}_j}\prod_{p\neq n,j}\delta_{\mathbf{k}_p'-\mathbf{k}_p}.$$

The matrix element of the unperturbed resolvent $R_0(z)$ is trivial:

$$\langle\{\mathbf{k}\}|R_0(z)|\{\mathbf{k}'\}\rangle = -\frac{1}{iz}\delta_{\{\mathbf{k}\}-\{\mathbf{k}'\}}. \tag{3.5}$$

The structure of Eqs. (3.3)-(3.5) permits a graphical representation using the same diagrams as in [8]. From (3.4) for the matrix element of the interaction operator, it follows that in our case (cf. [8]) there are three nontrivial single-vertex diagrams (see Fig. 1). The matrix elements corresponding to these are

$$\langle\mathbf{k}_n,\mathbf{k}_j|L_{nj}|\mathbf{k}_n'\rangle = 4\pi^2\Omega^{-1}\mathbf{e}\times i(\mathbf{k}_n'-\mathbf{k}_n)V(|\mathbf{k}_n'-\mathbf{k}_n|)\,i\mathbf{k}_n'\delta_{\mathbf{k}_n'-\mathbf{k}_n-\mathbf{k}_j}, \tag{C}$$

$$\langle\mathbf{k}_n,|L_{nj}|\mathbf{k}_n',\mathbf{k}_j'\rangle = 4\pi^2\Omega^{-1}\mathbf{e}\times i(\mathbf{k}_n'-\mathbf{k}_n)V(|\mathbf{k}_n'-\mathbf{k}_n|)\,i(\mathbf{k}_n'-\mathbf{k}_j')\,\delta_{\mathbf{k}_j'+\mathbf{k}_n'-\mathbf{k}_n}, \tag{D}$$

$$\langle\mathbf{k}_n,\mathbf{k}_j|L_{nj}|\mathbf{k}_n',\mathbf{k}_j'\rangle = 4\pi^2\Omega^{-1}\mathbf{e}\times i(\mathbf{k}_n'-\mathbf{k}_n)V(|\mathbf{k}_n'-\mathbf{k}_n|)\,i(\mathbf{k}_n'-\mathbf{k}_j')\,\delta_{\mathbf{k}_n'+\mathbf{k}_j'-\mathbf{k}_n-\mathbf{k}_j}. \tag{E}$$

From these relations and the representation (3.3) vertices C, D, and E have topological indices equal to 0, 1, and 0, respectively [8].

4. We consider the characteristic times for our model. Because we are modeling two-dimensional turbulence, we can take as characteristic randomization processes the intensities ε and η in the cascade processes [13]. The parameters $\varkappa$, c, ε, η give the dimensionless combination $\Gamma = c^{1/2}\eta^{-1/2}\varepsilon^{1/2}$. Hence we represent quantities with the dimensions of time in the form

$$T_m = \varepsilon^{1/2}(\varkappa^2 c\eta)^{-1/2}\Gamma^m. \tag{4.1}$$

With $m = -1$ we have $\tau_i = (\varkappa c)^{-1}$ from (4.1) and for $m = 0$ we have $\tau_r = (\varkappa^2 c)^{-1/2}\varepsilon^{1/2}\eta^{-1/2}$. The time τ_i is the circulation period of a pair of vortices with mean separation $c^{-1/2}$ and can naturally be identified with the characteristic interaction time. The time τ_r, involving the intensity of the cascade processes, can be considered as a characteristic relaxation time. If $\eta \sim \varepsilon \sim O(1)$ it follows from (2.2) that $\tau_i \ll \tau_r$. But in real turbulence these times can be of the same order. In our case for interactions determining convective transport and relaxation from (3.3), only the form of the dependence of τ_i and τ_r on $\varkappa$, c is needed.

We show that in order to derive (2.1) it is necessary to select from (3.3) and sum all contributions in $\rho(\mathbf{k}.\ t)$ proportional to $(\varkappa c)^n$. As in [14] it can be shown that in this case all quantities of order $(t/\tau_i)^n/n!$ are accounted for.

The contributions are given by all possible diagrams containing only vertices of type D and having one outer line to the left. Transforming from the diagram series of Fig. 2 to the corresponding mathematical expression (3.3) gives

$$\begin{aligned}\rho(\mathbf{k}_n,t) = {} & (2\pi)^{-1}\oint dz e^{-izt}\frac{1}{iz}\rho(\mathbf{k}_n,0) + \oint dz e^{-izt}\frac{1}{iz}(-\varkappa)\times\\ & \times\sum_{\mathbf{k}_j'}\sum_{j=1}^{N}\langle\mathbf{k}_n|L_{nj}|\mathbf{k}_n',\mathbf{k}_j'\rangle\left\{\frac{1}{-iz}\left[\rho(\mathbf{k}_n',0)\rho(\mathbf{k}_j',0) + (-\varkappa)\sum_{\mathbf{k}_m''}\sum_{m=1}^{N}\langle\mathbf{k}_n',k_j'\right.\right.\\ & \left.\left.|L_{nm}|\mathbf{k}_n'',\mathbf{k}_m''\rangle\frac{1}{-iz}\rho(\mathbf{k}_n'',0)\rho(\mathbf{k}_j',0)\rho(\mathbf{k}_m'',0) + \ldots\right]\right\}.\end{aligned} \tag{4.2}$$

In order to obtain contributions of the required order, we include on the right-hand side only the factorized parts of the Fourier coefficients [8].

After differentiation with respect to t, the first term in (4.2) goes to zero according to the Cauchy theorem. In the second term the first vertex operator is taken outside of the integral sign and the remaining expression is integrated to give a contribution of order $(\varkappa c)^n$ in $\rho(\mathbf{k}_n',\ t)\ \rho\ (\mathbf{k}_j',\ t)$. Taking the limit $N \to \infty$, $\Omega \to \infty$, $N/\Omega \to c$ we obtain the following equations for the Fourier transforms:

$$\frac{\partial\rho(\mathbf{k},t)}{\partial t}=4\pi^2\varkappa c\mathbf{e}\times\int d\mathbf{k}_1 i(\mathbf{k}_1-\mathbf{k})V(|\mathbf{k}-\mathbf{k}_1|)\,i(2\mathbf{k}_1-\mathbf{k})\,\rho(\mathbf{k}_1,t)\,\rho(\mathbf{k}-\mathbf{k}_1,t). \tag{4.3}$$

The original variables are recovered using the inverse Fourier transform $\int d\mathbf{k}e^{i\mathbf{k}\mathbf{r}}\rho(\mathbf{k},t)=c^{-1}[f(\mathbf{r},t)-c]\equiv n(\mathbf{r},t)$. With the help of the substitution

$$\mathbf{k}-\mathbf{k}_1=\mathbf{k}_2,\ \delta(\mathbf{k}_2-\mathbf{k}_3)=(4\pi^2)^{-1}\int d\mathbf{r}e^{i(\mathbf{k}_3-\mathbf{k}_2)\mathbf{r}_1}$$

the right-hand side of (4.3) can be written in the form

$$-\varkappa c\mathbf{e}\times\int d\mathbf{r}_1\left[\int d\mathbf{k}_2 e^{i\mathbf{k}_2(\mathbf{r}-\mathbf{r}_1)}i\mathbf{k}_2V(k_2)\right]\left[\int d\mathbf{k}_1 e^{i\mathbf{k}_1\mathbf{r}}i\mathbf{k}_1\rho(\mathbf{k}_1,t)\right]\left[\int d\mathbf{k}_3 e^{i\mathbf{k}_3\mathbf{r}_1}\rho(\mathbf{k}_3,t)\right].$$

We thus arrive at the equation

$$\frac{\partial f(\mathbf{r},t)}{\partial t}=-\varkappa c\mathbf{e}\times\nabla\int d\mathbf{r}_1V(|\mathbf{r}-\mathbf{r}_1|)\,n(\mathbf{r}_1,t)\,\nabla f(\mathbf{r},t),$$

which is identical to (2.1) except for the factor c.

As is well known, a self-consistent field equation cannot describe relaxation processes. Therefore, in addition to (4.2) we consider all possible contributions to $\rho(\mathbf{k},t)$ from "pseudodiagonal" [8] fragments (Fig. 3).

Formal estimates of the time dependence of these contributions show that after summation quantities of order

$$\sum_n\frac{1}{n!}(t/\tau_r)^{2n}, \tag{4.4}$$

are accounted for, which are important over times of order $t\sim\tau_r$. However, the procedure used above for the derivation of the closed equation for $\rho(\mathbf{k},t)$ leads in this case to the appearance on the right-hand side in (4.3) of an additional non-Markovian term of the form

$$\int_0^t d\theta G_0(t-\theta)\,\rho(\mathbf{k},\theta),$$

where $G_0(\theta)$ does not depend on time. This implies that the vortex interaction time is unbounded and leads to a divergence when $t\to\infty$. Again the equation does not describe relaxation processes because the additional term is reversible in time, as is clear directly from (4.4) where only even powers of t occur.

5. The divergence is the consequence of the attempt to describe a strongly interacting system by a sequence of binary processes separated in time. The divergence can be suppressed if the collective nature of the vortex interactions is taken into account. This can be done by renormalizing the propagation function (propagator) [9], which in this case is described by a closed equation.

We consider the diagrammatic representation of the renormalized propagator in Fig. 4. Here we take into account contributions from all possible diagonal fragments formed by the introduction of the elementary diagrams of Fig. 3a, b between vertices of types D and C.

The analytical expression for the operator series corresponding to Fig. 4 is given by

$$G(\mathbf{k},t)=(2\pi)^{-1}\oint dze^{-izt}\frac{1}{-iz}\sum_{n=0}^{\infty}\left[\Phi(\mathbf{k},z)\frac{1}{-iz}\right]^nG(\mathbf{k},0),\ G(\mathbf{k},0)=I, \tag{5.1}$$

where I is the identity operator and

$$\Phi(\mathbf{k},z)=\sum_{m=0}^{\infty}(-\varkappa)^{2(m+1)}\langle\mathbf{k}|L_D[R_0(z)L]^{2m}R_0(z)L_C|\mathbf{k}\rangle,$$

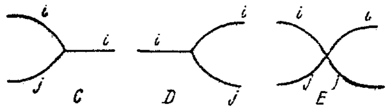

Fig. 1

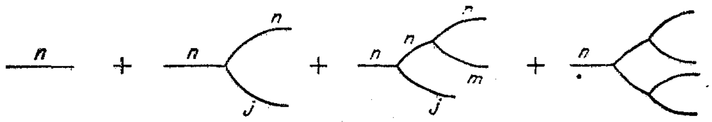

Fig. 2

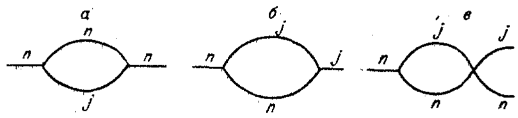

Fig. 3

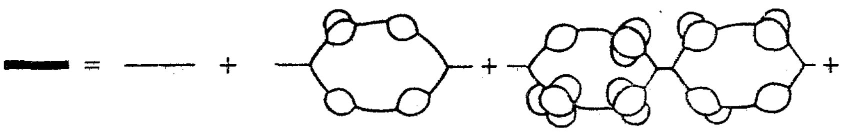

Fig. 4

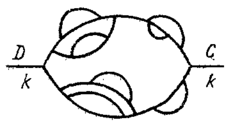

Fig. 5

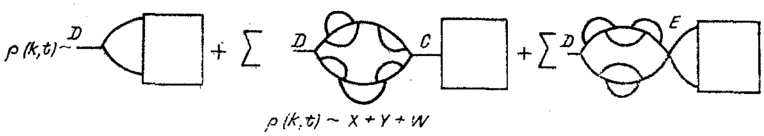

Fig. 6

i.e., $\Phi(\mathbf{k}, z)$ is an infinite sum of all possible diagrams (Fig. 5).

After differentiation of (5.1) with respect to time using the representation of the propagator (5.1) and the convolution theorem for the Laplace transform we obtain

$$\frac{\partial G(\mathbf{k},t)}{\partial t} = (2\pi)^{-1}\oint dz e^{-izt}\Phi(\mathbf{k},z)\frac{1}{-iz}\sum_{n=0}^{\infty}\left[\Phi(\mathbf{k},z)\frac{1}{-iz}\right]^n G(\mathbf{k},0) = \int_0^t d\theta Z(\mathbf{k},t-\theta)\,G(\mathbf{k},\theta); \tag{5.2}$$

$$Z(\mathbf{k},t) = (2\pi)^{-1}\oint dz e^{-izt}\,\Phi(\mathbf{k},z). \tag{5.3}$$

We transform the operator series of (5.3) with the help of the factorization theorem of Resibois [15]. Each diagram of Fig. 5 can be separated into two independent branches by discarding the terminal D and C vertices. The diagrams entering $\Phi(\mathbf{k}, z)$ are grouped into classes, each of which contains all diagrams having a fixed number of vertices in each branch and obtained by all possible permutations of vertices of one branch with respect to vertices of the other and conserving their order of appearance in the branches. According to the theorem of Resibois the contribution of each class is equal to that of one of the diagrams in which the contribution of the internal part is given by the product of the contributions of the independent branches. Summing over the factorized contributions from all classes we find from the structure Fig. 5 entering $\Phi(\mathbf{k}, z)$, the sum of contributions of the branches is given by the product of propagator contributions (Fig. 4). Therefore series (5.3) can be written in the form

$$Z(\mathbf{k}_i, t) = \sum_{j=1}^{N} \sum_{\mathbf{l}} (-\varkappa)^2 \langle \mathbf{k}_i | L_{ij} | \mathbf{k}_i - \mathbf{l}_i, \mathbf{l}_j \rangle G(\mathbf{k}_i - \mathbf{l}_i, t)\ G(\mathbf{l}_j, t) \langle \mathbf{k}_i - \mathbf{l}_i, \mathbf{l}_j | L_{ij} | \mathbf{k}_i \rangle.$$

After taking the limit to infinite space, we obtain the following equation for the renormalized propagator

$$\frac{\partial G(\mathbf{k}, t)}{\partial t} = -4\pi^2\varkappa^2 c \int_0^t d\theta P(\mathbf{k}, t-\theta) G(\mathbf{k}, \theta), \qquad (5.4)$$

$$P(\mathbf{k}, \theta) = \int d\mathbf{l}\, [\mathbf{el}\,(\mathbf{k} - 2\mathbf{l})\, V^2(l)\, \mathbf{elk}]\, G(\mathbf{k} - \mathbf{l}, \theta)\, G(\mathbf{l}, \theta).$$

We consider now all possible contributions of order $(\varkappa c)^n(\varkappa^2 c)^m$, $n, m = 0, 1, \ldots$ in $\rho(\mathbf{k}, t)$. The diagrammatic representation is given in Fig. 6. Here the rectangles denote the infinite sum of all possible paths consisting of vertices of type D and the "pseudo-diagonal" fragments of Fig. 3. In the second and third groups the outer sums are taken over all possible "leading" diagrams. The general features of the derivation of the equation for $\rho(\mathbf{k}, t)$ follow the derivation of the equation in Sec. 4. The second and third groups are transformed using the Resibois theorem and the convolution theorem for the Laplace transform as in the derivation of (5.2) and (5.4). This is made clear if we note that an analytical expression equivalent to Fig. 6 is

$$\rho(\mathbf{k}, t) = \oint dz e^{-izt} \sum_{m=0}^{\infty} \frac{1}{-iz}\left[\Phi(\mathbf{k}, z)\frac{1}{-iz}\right]^m \left[\rho(\mathbf{k}, 0) + \sum_{\{\mathbf{k}'\}} D_{\mathbf{k}\{\mathbf{k}'\}}\right]$$

and

$$Y = \oint dz e^{-izt} \sum_{m=1}^{\infty} \frac{1}{-iz}\left[\Phi(\mathbf{k}, z)\frac{1}{-iz}\right]^m \left[\rho(\mathbf{k}, 0) + \sum_{\{\mathbf{k}'\}} D_{\mathbf{k}\{\mathbf{k}'\}}\right],$$

where $D_{\mathbf{k}\{\mathbf{k}'\}}$ is a generalized notation representing all contributions with vanishing correlations.

The equation for $\rho(\mathbf{k}, t)$ takes the form

$$\frac{\partial \rho(\mathbf{k},t)}{\partial t} = 4\pi^2\varkappa^2 c\mathbf{e} \times \int d\mathbf{k}_1 i(\mathbf{k}_1 - \mathbf{k}) V(|\mathbf{k} - \mathbf{k}_1|)\, i(2\mathbf{k}_1 - \mathbf{k})\, \rho(\mathbf{k}_1, t) \times \qquad (5.5)$$

$$\times \rho(\mathbf{k} - \mathbf{k}_1, t) - 4\pi^2\varkappa^2 c \int_0^t d\theta P(\mathbf{k}, \theta)\, \rho(\mathbf{k}, t-\theta) - 4\pi^2\varkappa^2 c \int_0^t d\theta \int d\mathbf{k}_1 P_1(\mathbf{k},$$

$$\mathbf{k}_1, \theta)\, \rho(\mathbf{k}_1', t-\theta)\, \rho(\mathbf{k} - \mathbf{k}_1', t-\theta),$$

$$P_1(\mathbf{k}, \mathbf{k}_1, \theta) = \int d\mathbf{l}\mathbf{e} \times \mathbf{l} V(l)(\mathbf{k} - 2\mathbf{l})\, G(\mathbf{k} - \mathbf{l}, \theta)\, G(\mathbf{l}, \theta)\, \mathbf{e} \times (\mathbf{k} - \mathbf{l})\, V(|\mathbf{k}_1 - \mathbf{l}|) \times (2\mathbf{k}_1 - \mathbf{k}).$$

Equations (5.5) and (5.3) are non-Markovian and give closed descriptions of vortex evolution over times of order τ_r.

We take the Markovian limit of (5.5) by first constructing an approximate solution to (5.3). We will put $|\mathbf{k}| \sim |l|$ so that the characteristic size of the nonuniformities is close to the effective interaction length; this corresponds to the assumption made in the phenomenological theory of turbulence. Writing $G(\mathbf{k}, t) = G(\alpha kt)$ we can make the substitution $G(\mathbf{l}, \theta) \simeq G(\mathbf{k}, \theta)$, $G(\mathbf{k} - \mathbf{l}, \theta) \cong I$. in (5.3), which then becomes an integral equation of convolution type

$$\frac{\partial G(\mathbf{k}, t)}{\partial t} = -\alpha^2\mathbf{k}^2 \int_0^t d\theta G(\mathbf{k}, t-\theta)\, G(\mathbf{k}, \theta), \quad \alpha^2 = 4\pi^2\varkappa^2 c a^2, \qquad (5.6)$$

$$a^2 = \int d\mathbf{l} V^2(l)\, [1 - \cos^2(\mathbf{k}, \mathbf{l})]\, l^2, \ a^2 > 0.$$

Solving this with the help of the Laplace transform we find [16]

$$G(\mathbf{k},\ t) = (\alpha kt)^{-1} J_1(2\alpha kt), \tag{5.7}$$

where $J_1(x)$ is the first-order Bessel function of the first kind. It is seen from (5.7) that the form of the propagator is as discussed above. When $t \to 0$ we have $G(\mathbf{k}, t) \to I$.

The Markovian limit to (5.5) is taken in the usual way [14]. It is necessary to compute contributions given by products of propagators. Again putting $|\mathbf{k}| \sim |l|$ we have [17]

$$\int_0^\infty d\theta G(\mathbf{k}-\mathbf{l}, \theta) G(\mathbf{l},\ \theta) = (\alpha^2 l)^{-1} |\mathbf{k}-\mathbf{l}|^{-1} \int_0^\infty d\theta J_1(2\alpha|\mathbf{k}-\mathbf{l}|\theta) \times$$
$$\times J_1(2\alpha l\theta) = \frac{1}{2\alpha(l+|\mathbf{k}-\mathbf{l}|)} \frac{\Gamma(1/2)}{\Gamma(2)\Gamma(3/2)} F\left[1/2, 3/2, 3, \frac{4l|\mathbf{k}-\mathbf{l}|}{(l+|\mathbf{k}-\mathbf{l}|)^2}\right] \simeq \frac{1}{\alpha l}, \tag{5.8}$$

where $\Gamma(x)$ is the gamma function and $F(\alpha, \beta, \gamma, z)$ is the Gaussian hypergeometric function.

Using (5.8) we perform the inverse Fourier transform in (5.5). The intermediate steps are analogous to those used in (4.3). The second and third terms in (5.5) lead to the forms

$$-4\pi^2\varkappa^2 c \int d\mathbf{k} e^{i\mathbf{kr}} \ldots = -(\varkappa^2 c)^{1/2} a_1 a^{-1} \int d\mathbf{k} e^{i\mathbf{kr}} k^2 \rho(\mathbf{k}, t); \tag{5.9}$$

$$-4\pi^2\varkappa^2 c \int d\mathbf{k} e^{i\mathbf{kr}} \ldots = -(\varkappa^2 c)^{1/2} a^{-1} \int d\mathbf{r}_1 \Big\{ \int d\mathbf{k}_2 e^{i\mathbf{k}_2(\mathbf{r}-\mathbf{r}_1)} \mathbf{e} \times \tag{5.10}$$
$$\times i\mathbf{k}_2 V(k_2) \mathbf{C}(\mathbf{r}-\mathbf{r}_1) \int\int d\mathbf{k} d\mathbf{k}_1 e^{i\mathbf{kr}} e^{i\mathbf{k}_1\mathbf{r}_1} (i\mathbf{k}_1 i\mathbf{k}_1 - i\mathbf{k} i\mathbf{k}) \rho(\mathbf{k}, t) \rho(\mathbf{k}_1, t) \Big\}.$$

In (5.9) a_1 is defined similarly to a in (5.6) and differs only by the additional factor l^{-1} in the integrand. In order to calculate

$$\mathbf{C}(\mathbf{r}-\mathbf{r}_1) = \mathbf{e} \times \int d\mathbf{l} e^{i\mathbf{l}(\mathbf{r}-\mathbf{r}_1)} i\mathbf{l} l^{-1} V(l) = -2\pi \frac{(\mathbf{r}-\mathbf{r}_1)}{|\mathbf{r}-\mathbf{r}_1|} \int_0^\infty dl l V(l) J_1(l|\mathbf{r}-\mathbf{r}_1|)$$

an explicit expression is needed for $V(l)$. Here we use the generalized Fourier transform of the potential $V(r) = \ln r$:

$$V(l) = \lim_{\beta\to 0} (4\pi^2)^{-1} \int d\mathbf{r} e^{-i\mathbf{lr}} e^{-\beta r} \ln r = -(2\pi l^2)^{-1}. \tag{5.11}$$

Then we have

$$\mathbf{C}(\mathbf{r}-\mathbf{r}_1) = \mathbf{e} \times \nabla|\mathbf{r}-\mathbf{r}_1|. \tag{5.12}$$

Note that the use of (5.11) in the calculation of (5.6) leads to an improper integral which is logarithmically divergent at its upper and lower limits. A corresponding divergence $O(l^{-1})$ occurs in a_1 at the lower limit.

The divergence as $l \to 0$ is due to the collective nature of the vortex interactions. The divergence can be avoided if in place of $V(r)$ we use an approximation. For example if the vortex temperature [18] is positive a satisfactory approximation is given by the MacDonald function $K(r)$, which is the two-dimensional analog of the Debye–Hooker potential.

The divergence at $l \to \infty$ is due to the fact that a point model of the vortices is not applicable at small distances. To remove this divergence one may either refine (i.e., complicate) the model [5] or simply cut off the region of integration at some value $l = l_{max}$. Because explicit forms for a and a_1 are not used in our approach, we will simply assume below that they are bounded.

From (5.9) to (5.12) equations for the original variables are written in the form

$$\frac{\partial f(\mathbf{r}, t)}{\partial t} + \mathbf{U}(\mathbf{r}, t)\nabla f(\mathbf{r}, t) = (\varkappa^2 c)^{1/2} a_1 a^{-1} \Delta f(\mathbf{r}, t) + (\varkappa^2 c)^{1/2} a^{-1} \times \tag{5.13}$$
$$\times \int d\mathbf{r}_1 n(\mathbf{r}_1, t) \mathbf{B}(\mathbf{r}-\mathbf{r}_1) : \nabla\nabla f(\mathbf{r}, t) - (\varkappa^2 c)^{1/2} a^{-1} n(\mathbf{r}, t) \int d\mathbf{r}_1 \mathbf{B}(\mathbf{r}-\mathbf{r}_1) : \nabla_1\nabla_1 f(\mathbf{r}_1, t),$$

where $\mathbf{B}(\mathbf{r}-\mathbf{r}_1) = e \times \nabla|\mathbf{r}-\mathbf{r}_1| e \times \nabla (|\mathbf{r}-\mathbf{r}_1|)$ is a second rank tensor and the colon denotes the tensor scalar product.

6. We consider some qualitative properties of our equation. Since a, $a_1 > 0$, the first term on the right-hand side of (5.13) containing the Laplacian gives a positive diffusion of the vortex distribution. The components of the tensor $B(\mathbf{r}-\mathbf{r}_1)$ form the quantity

$$(\mathbf{B}\xi, \xi) = e_{ik} \frac{\partial |\mathbf{r}-\mathbf{r}_1|}{\partial x_k} e_{jn} \frac{\partial V(|\mathbf{r}-\mathbf{r}_1|)}{\partial x_n} \xi_i \xi_j, \tag{6.1}$$

where $r = (x_1, x_2)$, e_{ij} is the antisymmetric tensor ($e_{12} = -e_{21} = 1$, $e_{11} = e_{22} = 0$ and a summation from 1 to 2 is implied over repeating indices. It can be verified directly that the form (6.1) is nonnegative. Using this, it can be shown for a continuous $n(\mathbf{r}, t)$ that the first and second terms on the right-hand side of (5.13) define an elliptic operator of second order whose sign depends on the distribution $f(\mathbf{r}, t)$ and can be different at different points in the fluid. The sign of the last term is given by the local value of $n(\mathbf{r}, t)$.

This means that (5.13) can describe local decay processes (positive diffusion) as well as creation processes (negative diffusion) of the large-scale vortex structure. As is well known [19] the latter effect is identified with the phenomenon of negative viscosity.

It can also be shown that over the entire fluid the solution (5.13) relaxes to a stationary distribution. We consider the entropy production

$$S(t) = -\int d\mathbf{r} f(\mathbf{r}, t) \ln f(\mathbf{r}, t) \tag{6.2}$$

during the evolution of the distribution. We will assume that when $r \to \infty$, $f(\mathbf{r}, t)$ decays sufficiently rapidly, along with its derivatives. We multiply (5.13) by $(1 + \ln f(\mathbf{r}, t)$ and integrate over all space. After some transformations, the equation for entropy balance takes the form

$$\begin{aligned} \frac{dS}{dt} = \int d\mathbf{r} \nabla \mathbf{U}(\mathbf{r}, t) f(\mathbf{r}, t) + (\varkappa^2 c)^{1/2} a_1 a^{-1} \int d\mathbf{r} [f(\mathbf{r}, t)]^{-1} [\nabla f(\mathbf{r}, t)]^2 + \\ + (1/2)(\varkappa^2 c)^{1/2} a^{-1} \iint d\mathbf{r} d\mathbf{r}_1 \nabla\nabla : \mathbf{B}(\mathbf{r}-\mathbf{r}_1) [\ln f(\mathbf{r}, t) - \ln f(\mathbf{r}_1, t)] [f(\mathbf{r}, t) - \\ - f(\mathbf{r}_1, t)] + (\varkappa^2 c)^{1/2} a^{-1} \iint d\mathbf{r} d\mathbf{r}_1 \mathbf{B}(\mathbf{r}-\mathbf{r}_1) : \nabla_1 f(\mathbf{r}_1, t) \nabla_1 f(\mathbf{r}_1, t) [f(\mathbf{r}, t)]^{-1} \times \\ \times n(\mathbf{r}, t) + (\varkappa^2 c)^{1/2} a^{-1} \iint d\mathbf{r} d\mathbf{r}_1 \nabla\nabla : \mathbf{B}(\mathbf{r}-\mathbf{r}_1) f(\mathbf{r}_1, t) n(\mathbf{r}, t). \end{aligned} \tag{6.3}$$

In order to integrate the last two terms in (5.13) by parts, a symmetrization is performed.

Since the fluid is incompressible $\nabla U(\mathbf{r}, t) = 0$ and the convective terms gives zero contribution to the entropy production. The positive contribution of the second term on the right-hand side of (6.3) is obvious. The third term is also positive-definite because $\nabla\nabla : B(\mathbf{r}-\mathbf{r}_1) = |\mathbf{r}-\mathbf{r}_1|^{-3} \geqslant 0$ and we have the inequality $[\ln f(\mathbf{r}, t) - \ln f(\mathbf{r}_1, t)][f(\mathbf{r}, t) - f\ \mathbf{r}_1, t)] \geqslant 0$.

Because (6.1) and (6.4) are positive-definite, the sign of the last two integrals in (6.3) depends only on the behavior of the function $n(\mathbf{r}, t)$. From the quasineutrality condition $d\mathbf{r} f(\mathbf{r}, t) = 0$ and the assumed nature of the decay of $f(\mathbf{r}, t)$ in the limit $r \to \infty$, it follows that negative contribution of these integrals will be confined to the outer regions of the flow where the other parts of the integrands go to zero rapidly. It then follows that the total contribution of these integrals in the entropy production will either be nonnegative or at worst, close to zero in absolute value.

Thus, for the evolution of $f(\mathbf{r}, t)$ satisfying (5.13), the entropy (6.2) monotonically (in a general sense) increases. On the other hand, from the convergence of the integral $\int d\mathbf{r} f(\mathbf{r}, t) < \infty$ it can easily be shown [20] that the functional (6.2) is bounded. Thus in the evolution process the solution (5.13) goes to a stationary distribution.

Analysis of the results of numerical experiments [2, 4] with conditions close to the assumptions made here show that (5.13) correctly gives the evolution of a large system of point vortices confined to a plane.

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STATISTICAL PROPERTIES OF BURSTS OF TURBULENT FLUCTUATIONS

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The energy spectra, probability distribution functions (DFs), and the associated moment and scale numerical characteristics are used to describe turbulent fluctuations at a certain point of a flow in statistical fluid mechanics. However, these functions do not describe the instantaneous disturbances generated in turbulent flows; these disturbances are particularly important in a number of engineering applications.

An alternative approach to the investigation of turbulence is possible, consisting in the analysis of bursts, i.e., events where the fluctuation component of the flow velocity exceeds a certain prescribed level. Apart from practical applications, the burst characteristics determined by the joint distribution of the probabilities of the fluctuation velocity of the flow and its derivative are important from the standpoint of methods being developed at the present time for the description of turbulent flows on the basis of the DF equations. This kind of approach can be used in studying the laminar-to-turbulent flow transition, which is characterized by the inception of randomly distributed local regions with large gradients of the parameters.

The theory of bursts of stochastic processes, which was formulated primarily for radiophysical applications (see, e.g., [1]), is currently in a state of continuing development.

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